A Model for Trade-Up and Change in Considered Brands

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Abstract

A common theme in the marketing literature is the acquisition and retention of customers as they trade-up from inexpensive, introductory offerings to those of higher quality. Standard models of choice, however, apply to narrowly defined categories for which assumptions of near-perfect-substitution are valid. We extend the non-homothetic choice model of Allenby and Rossi (1991) to accommodate effects of advertising, professional recommendation and other factors that facilitate the description and management of trade-up. The model is applied to a national study of an over-the-counter health product.

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1. Introduction

In many product categories, a wide array of products of differing quality and price are offered. Examples include automotive and electronic products, and also what appear to be narrowly defined categories such as razor blades where a wide variety of price points and qualities from inexpensive disposables to three or four blade shaving "systems" are displayed. In marketing research, it is common to narrow consideration to a set of products that are highly substitutable rather than to model an entire category. In part, this is due to the limitations of commonly used models such as logit models that assume a very high degree (if not, perfect) substitutability¹ between products. In a logit or probit model, changes in overall category expenditure do not affect choice probabilities due to the assumption of a homothetic utility structure. In marketing practice, we do not want to limit attention to narrow sub-categories but, instead, desire a choice model which is applicable to a board category of items of diverse quality and price.

Our data comes from an over-the-counter health care category. In this category, the manufacturer uses pricing, professional recommendation and advertising to manage the entire product line. Given that higher quality products have higher margins, it is important to consider the effect of marketing actions on the decision of consumers to trade-up and allocate higher expenditure to the product category.

Trade-up occurs when a consumer makes a jump from an inferior to a superior good within a product category. The trade-up event is often associated with a change in life-stage,

¹ We note that, in theory, random coefficient logit models can introduce more flexibility in substitution patterns. However, these models cannot remove the assumption of homotheticity which we focus on here.

personal priorities or in disposable income. Our interest is in the trade-up decision and its relationship to variables under our control, such as advertising and professional recommendations, and those that are not, such as age and income. Since these same variables can also influence the budgetary allotment made by consumers, as well as simple preference, it is important to formally model the consumer decision to disentangle their effects.

Standard discrete choice models, such as the logit model and probit model, assume constant marginal utility that leads to linear indifference curves and corner solutions. These models are easy to estimate and are the widely employed in models of consumer demand, both at the aggregate and disaggregate levels (c.f. Train (2003)). However, the assumption of constant marginal utility implies a homothetic utility structure where changes in the budgetary allotment do not affect the probability that a specific alternative is chosen.

To allow for analysis of a group of products that are not perfect substitutes, we extend the non-homothetic utility function of Allenby and Rossi (1991) to accommodate the effects of advertising and professional recommendation, the presence of an outside good, and product attributes. The non-homothetic function retains the property of linear indifference curves (assumed in the standard logit/probit formulation), ensuring corner solutions, while allowing marginal utilities to change as overall expenditure increases. Indifference curves fan out in the positive orthant, with their rates of rotation related to consumer tendency to trade-up to higher quality offerings if their budget allows. Our model separates the effects of baseline preferences, trade-up, and changes in the considered set of brands as budgetary allotments increase.

Our non-homothetic model provides a new manner in which advertising may affect demand. In standard logit models, advertising can only affect brand intercepts (this is true even in the dynamic learning models of Erdem and Keane (1996) – in these models advertising

provides a signal of the value of the brand intercept). In our non-homothetic model, advertising can have an effect on the marginal willingness to pay for the quality attribute. That is, advertising can accentuate the motive for trading up from lower quality to higher quality brands.

We apply our model to a nationally representative sample of consumers using a virtual shopping experiment that portrays actual retail shelf layouts. Respondents engage in choice tasks prior to and after viewing advertisements for a new benefit available in some of the higher quality offerings. The pre-post measurement of demand under different prices, and background information provided by the respondents, provides sufficient information for understanding the effects of professional recommendations that were made prior to the experiment, and the effect of advertising during the experiment. We find a strong relationship of these effects to respondents' age and income.

The results provide a description of consumer behavior useful for effectively targeting marketing expenditures for trade-up. We find that perceptions of quality differences among choice alternatives to be most pronounced among young, high-income respondents. As respondents become older, their aspiration brands appear to lose their appeal as the perceived quality differences among brands is less pronounced. Our results contribute to the marketing discipline's understanding of factors associated with trade-up. We also investigate the management of trade-up through the use of profession endorsements and media advertising.

The remainder of this paper is organized as follows. Section 2 presents the nonhomothetic choice model for trade-up, and contrasts its properties to standard discrete choice models. The data and choice experiment is described in section 3, and section 4 presents estimation results. Section 5 then investigates use of the model for understanding and managing consumer trade-up. Concluding remarks are offered in section 6.

2. A Trade-Up Model

An analysis of trade-up behavior requires a model with three properties: affordability, superiority and baseline preference. Affordability is modeled through the budgetary allotment a consumer assigns to an extended product category that includes an unknown, outside good. This allotment can be allocated to the purchase of an available item with the remainder saved for future use. Thus, we assume that respondents are maximizing their utility across items in the product category, represented by the vector x, and an outside good, z, subject to a budget constraint:

$$\max \ln u(x,z) = \ln u(x) + \tau \ln(z) \quad s.t. \ p'x + z \le E$$
(1)

where p is the vector of prices for the choice alternatives, E is the budget allotment, and the price of the outside good is assumed to be \$1.00. If the price of an item, p_i , is greater that the allotment, E, then the utility maximizing solution must assign $x_i = 0$.

Superiority is modeling using the non-homothetic utility function of Allenby and Rossi (1991) with an outside good. The utility for the vector of demand, x, is defined implicitly as:

$$u(x) = \sum_{k=1}^{K} \psi_k(\overline{u}) x_k = \sum_{k=1}^{K} \exp\left[\alpha_k - \kappa_{k^*} \overline{u}(x, z)\right] x_k$$
(2)

where the marginal utility of an offering, $\psi_k(\overline{u})$, is a function of attainable utility \overline{u} . Thus, as respondents allocate greater expenditure, attainable utility increases and marginal utilities change. The utility function has linear indifference curves for any fixed value of \overline{u} .

The parameter κ affects the rate of rotation of the indifference curves. As attainable utility increases, the ratio of marginal utilities, $\psi_i(\bar{u})/\psi_j(\bar{u}) = \exp\left[\alpha_i - \alpha_j + \bar{u}(\kappa_j - \kappa_i)\right]$, is increasing in attainable utility \bar{u} for $\kappa_i < \kappa_j$, implying that alternative i is superior to alternative j. Superior choice alternatives are defined as those associated with smaller values of κ , while alternatives with larger κ values are relatively inferior. The effect of price changes can be decomposed into substitution and income effects, where the income effect favors the superior good. Thus, the model has the property that price changes of superior will draw disproportional share from inferior offerings.

The indifference curves rotate in the positive orthant but cannot intersect for equation (2) to be a valid utility function. A necessary and sufficient condition for non-overlapping indifference curves is for all intercepts of the indifference curves to monotonically increase as utility increases, i.e., $\partial x_i / \partial \overline{u} > 0$. This condition is assured if $\kappa > 0$, and we enforce positive κ parameters during estimation by substituting $\kappa = \exp[\kappa^*]$ and estimating κ^* unrestricted.

Baseline preferences are modeled through the intercepts (α) in equation (2). Budgetary effects can be nullified in the model when the parameter κ is equal to zero for all alternatives. When this occurs, the utility function in (2) reverts to a standard discrete choice model, similar to that used in standard logit and probit analysis. Equation (2) therefore nests common homothetic utility specifications.

We employ equations (1) and (2) to study choices among a large set of alternatives, with some prices ten times greater than others, indicating that they are not near-perfect substitutes. Since the indifference curves in the sub-utility function in equation (2) are linear, the utility maximizing solution will have just one choice alternative with nonzero demand. We can therefore engage in a direct search for the utility maximizing solution without resorting to the use of Kuhn-Tucker conditions (see Kim, Allenby and Rossi 2003).

The log utility associated with choosing one unit of alternative k is:

$$\ln u \left(x_k = 1, z_k \right) = \alpha_k - \kappa_k \overline{u}^k + \tau \ln \left(E - p_k \right)$$
(3)

where \overline{u}^k is the solution to the implicit equation:

$$\ln \overline{u}^{k} = \alpha_{k} - \kappa_{k} \overline{u}^{k} + \tau \ln \left(E - p_{k} \right)$$
(4)

which is obtained using numerical methods (e.g., Newton's method). The utility maximizing solution corresponds to the alternative (k) that maximizes the value of log utility in equation (3).

The likelihood for the data is obtained by introducing random error assumed to be known to the respondent and unobserved by the researcher. We introduce an additive error in equation (4) that allows us to nest alternative models that are restricted versions of our non-homothetic specification. Setting κ =0 corresponds to a homothetic specifications (e.g., logit and probit models), and setting τ =0 removes the restriction that the prices of considered brands must be less than the budgetary allotment. In this latter specification (κ =0, τ =0), the utility maximizing solution corresponds to the Kuhn-Tucker condition where ψ_k/p_k , or α_k – ln p_k , is maximum, i.e., a standard discrete choice specification.

The probability of selecting alternative k is:

$$\Pr(x_k = 1) = \Pr(\alpha_k - \kappa_k \overline{u}^k + \tau \ln(E - p_x) + \varepsilon_k > \alpha_i - \kappa_i \overline{u}^i + \tau \ln(E - p_i) + \varepsilon_i \quad \forall i \mid p_i \le E)$$
(5)

and assuming type I extreme value errors leads to choice probabilities of the form:

$$\Pr\left(x_{k}=1\right) = \frac{\exp\left[\alpha_{k} - \kappa_{k}\overline{u}^{k} + \tau\ln\left(E - p_{k}\right)\right]}{\sum_{\{i|p_{i} \leq E\}} \exp\left[\alpha_{i} - \kappa_{i}\overline{u}^{i} + \tau\ln\left(E - p_{i}\right)\right]}$$
(6)

Equation (6) reflects the affordability property of trade-up by assigning non-zero probability to alternatives that are within budget. Superiority is reflected in the κ parameters that govern the rates of rotation of the indifference curves. Baseline preference is reflected through α .

Setting all κ 's to zero results in a homothetic model with affordability and baseline preference:

$$\Pr\left(x_{k}=1\right) = \frac{\exp\left[\alpha_{k} + \tau \ln\left(E - p_{k}\right)\right]}{\sum_{\{i \mid p_{i} \leq E\}} \exp\left[\alpha_{i} + \tau \ln\left(E - p_{i}\right)\right]}$$
(7)

And, removing the budgetary allotment (E) from the utility maximizing solution corresponds to a standard choice model that captures baseline preferences only:

$$\Pr(x_k = 1) = \frac{\exp[\alpha_k + \tau \ln(p_k)]}{\sum_i \exp[\alpha_i + \tau \ln(p_i)]}$$
(8)

For all three models (6) - (8), a no-purchase decision can be included in the model specification as a separate choice alternative. This is desirable in survey research when partial arrays of offerings are shown to respondents, and the preference for the no-purchase alternative is interpreted as reservation value needed for positive demand.

Incorporating Attribute Information and Modeling Advertising Effects

In our analysis of the data described below, we investigate various parameterizations of the model intercepts (α) and rotation (κ) parameters. A popular technique for dealing with the presence of many choice alternatives that arise in the study of trade-up is to project these model parameters onto an attribute space. We allow both the intercept (baseline utility) and rotation parameters to be a function of attributes.

$$\alpha = A\tilde{\alpha} \qquad and \quad \kappa^* = A\tilde{\kappa}^* \tag{9}$$

where A is of dimension $i \times j$ with i > j. This specification constrains the estimated model intercepts (α) and/or rotation parameters (κ^*) to lie within the subspace defined by the column vectors of the attribute matrix A. The potential advantage of this specification is a significant reduction in number of parameters requiring estimation. Our non-homothetic demand model offers some new opportunities for incorporating advertising effects. Exposure to an ad can either change the brand intercepts or "baseline" utility parameters, α , or the quality parameters, κ . Previous structural work on advertising effects was limited to the logit or homothetic specification in which advertising could only change brand intercepts. If advertising is designed to increase the perceived quality of one brand at the expense of others (a common objective of advertising), then our model can accommodate this via a decrease of the associated κ . This means that the brand that is the focus of the advertising which be perceived as relatively more "superior" or of a higher quality. In our application, we model advertising as changing the κ associated with a particular attribute of two of the higher end products. Again, the non-homothetic model will allow for a change in the strength of the trade-up utility incentive as a result of advertising. This is a fundamentally different concept that merely a change in the brand intercept.

3. Data and Statistical Specification

Data are obtained from a national survey conducted by a leading packaged goods manufacturer. The product category under study is populated by 40 nationally branded offerings and a discount house brand, across three sub-categories: discount, regular and premium. Unfortunately, the manufacturer that commissioned the collection of this data has not allowed us to specify the product category or the brands.

Regular prices for the brands ranged from a minimum of \$0.79 and a maximum of \$219.99. Quotas were imposed on the sample so that respondents were currently users in the product category, approximately 50% of the respondents were male, and there were twice as many respondents currently using one of the discount offerings as a regular or a premium

offering (i.e., 50%, 25%, 25%). These quotas ensured conformity to the target population. A total of 1323 respondents provided data for analysis.

Data were collected in three phases. The first phase involved three choice tasks in which respondents choose from among offerings arrayed on a computer screen to resemble a shelf layout in a retail setting. High-quality graphics were used to represent the actual packaging of the alternatives, with a price sticker immediately below each. Respondents could select an item, read actual product descriptions, rotate the item, obtain a close-up and view the package back. When the respondents were done examining the items of interest to them, they proceeded to a check-out screen where their choice was recorded.

Between the first and second phase of data collection, a random sample of respondents were exposed to two television commercials that described a new product benefit available only among two of the premium offerings with a specific attribute (attribute C). Approximately 28% of the respondents were exposed to the advertisement. Video information was depicted differently in the two commercials – one showing a "lifestyle" orientation for Product 40, and the other being "high-tech" for Product 37. The effect of the video treatment was incorporating into the model by allowing it to alter the effect-size of the focal attribute. This was accomplished by modifying the attribute matrix, A, for the second and third phases of the data for those respondents viewing the commercial, and allowed us to measure the interactive effect of commercial on the marginal utility of the attribute.

The second phase of data collection involved five pair-wise choice tasks, with a nochoice option, where offerings within a respondent's currently used subcategory were displayed. Respondents who currently used discount offerings selected primarily among discounted offerings, regular users selected primarily among regular offerings, and premium users among

premium offerings. Two choice options were graphically displayed on the computer screen with prices below each, and respondents were asked to make a selection of one of the two, or a no-choice option. The purpose of phase two data collection was to ensure that sufficient information was obtained about current respondent preferences and budget allotments.

The third phase of data collection was similar to the first phase, except that 12 choice tasks were presented to the respondent. The purpose of phase three data collection was to ensure that sufficient information was obtained across all offerings.

Respondents in the survey also providing answers to questions about their current use of the product, demographic information, and information about recommendations that have received about the benefits of using the premium offerings. The likelihood for each respondent i is specified by three factors related to each phase of the data:

$$\pi \left(Data_{i} \mid \alpha_{i}, \kappa_{i}, \gamma_{i}, \tau_{i}, A_{1,i}, A_{23,i} \right) = \pi \left(Data_{i,1} \mid \alpha_{i}, \kappa_{i}, \gamma_{i}, \tau_{i}, A_{1,i} \right) \prod_{j=2}^{3} \pi \left(Data_{i,j} \mid \alpha_{i}, \kappa_{i}, \gamma_{i}, \tau_{i}, A_{23,i} \right)$$

$$(10)$$

where "Data_i" comprise the three sets of responses described above, { α_i } are baseline preference parameters, { κ_i } are the trade-up parameters that govern the relative rates of rotation of indifference curves, { γ_i } are the affordability parameters in logarithmic form, i.e., $\gamma = \ln E$ in equation (1), and { τ_i } are the parameters for the outside good in equation (1). We describe the attribute matrices { $A_{1,i}$ } and { $A_{23,i}$ } further below. Heterogeneity is incorporating using a random-effect specification:

$$\pi(\alpha_i, \kappa_i, \gamma_i, \tau_i \mid \Delta, z_i, V_\beta) = Normal(\Delta z_i, V_\beta)$$
(11)

where z_i is a vector of descriptor variables for respondent i:

$$z_{i} = (1, recommendation_{i}, income_{i}, age_{i}, income_{i} \times age_{i})$$
(12)

where "recommendation" is recorded as a dummy variable, indicating if the respondent received a professional recommendation for the premium product. This recommendation was received by the respondent sometime in the past, prior to this study. We note that the recommendation is for the set of premium products rather than for specific products. The video mock ad exposure is designed, however, to promote particular high end products.

The attribute matrix, A, consists of indicator variables for : 1. The seven brands in the dataset, 2. Quality tiers (discount, regular, premium), 3. Three brand attributes (A,B,C) available in regular and premium offerings. A_1 is the attribute matrix for the first phase of data collection. A_{23} is the attribute matrix for the second and third phases and includes two additional columns that indicate whether or not the respondent saw the advertisement videos with attribute C. There are two columns as one of the advertisements was targeted at product 37 and one for product 40.

Figure 1 displays the count and proportion of respondents receiving the recommendation for various age and income groups. The left side of the figure displays the sample proportion of respondents falling into each age-income group, and the right side of the figure displays the proportion within each cell that received the recommendation. It appears that current practice is to make recommendations to individuals that are older and wealthier. An issue we explore below is whether this practice is optimal. We use higher color temperatures to indicate higher values of the variable that is displayed. That is, dark blue represents the lowest values with red corresponding to the highest value.

We should note that if recommendations were made purely on the basis of our observed demographics such as income and age, we can make a causal assessment of the effect of a recommendation on demand. If, however, the professionals making the recommendation make these recommendations as a function of preference parameters, then our non-experimental data

cannot be used to evaluate the pure effect of a recommendation. That is, if professionals advise people who like premium brands anyway, then we could see an "effect" of a recommendation even if the recommendation is, in fact, completely ineffective in stimulating demand. It is our view that it is unlikely that a health care professional making the recommendation would simply be trying to match products to preferences but, instead, is acting as an informed agent for the customer – attempting to maximize their health outcome from product consumption.

== Figure 1 ==

The income, age and income × age variables are mean centered so that the intercept can be interpreted as the mean of the random-effects distribution for respondents who have not received a recommendation. Income is divided by 100,000 and age is divided by 100 so that the resulting Δ coefficients have approximately the same scale.

Estimation is carried out using Bayesian MCMC methods. Initial conditions of the chain were varied to ensure convergence to a common posterior. A total of 40,000 iterations were executed, with the last 10,000 iterations used for parameter estimation. Details are provided in the appendix.

4. Parameter Estimates

Table 1 displays fit statistics for five variations of the model. The first two models are characteristics models that constrain intercepts (α) and rotation parameters (κ) to lie within the characteristics subspace. The last three models relax this restriction for the intercepts, but retain it for the rotation parameters. Models 1 and 3 are standard logit models (equation 8) that measures baseline preferences and respondent price sensitivity. Model 4 incorporates affordability as in equation (7) by fixing the rotation parameters at zero. Models 2 and 5 also

incorporate the effects of trade-up by estimating the rotation parameters ($\tilde{\kappa}^*$) constrained to lie within the attribute space. We report the expected log marginal density of model of the data as a measure of fit.

== Table 1 ==

The fit statistics indicate that the characteristics models provide a poor fit to the data relative to models with unique intercepts. We also find that the proposed model for trade-up (equation 6) provides better fit than the logit model, equation (8), and the model that incorporates affordability (equation 7). Thus, we find evidence for the uniqueness of each of the offerings, and the presence of preference, trade-up and affordability in respondent choices.

Table 2 reports parameter estimates for a portion of the coefficient matrix Δ for model 5, the best fitting model. Reported are posterior means and posterior standard deviations for the trade-up (κ), affordability (γ or E) and outside good (τ) parameters. Estimates of the baseline preference parameters are not reported because the names of the 40 brands in the study cannot be disclosed for proprietary reasons, and therefore a detailed discussion of brand-specific coefficients (α) is not possible.

The left side of table 2 lists the attributes used in the analysis. There are seven brand names associated with the 40 offerings. Three attributes are associated with the discount, regular and premium nature of the offerings, and three additional attributes describing specific features that are available in the regular and premium offerings only. Attribute C is the focal attribute described in the video, through which the sponsoring firm hopes to generate trade-up in the product category.

The column headings in table 2 are descriptors of the respondents in the study. Some respondents received a recommendation to buy a superior offering by an expert prior to the

study. Respondent income, age, and an income by age interaction complete the description of the respondents. The remaining columns indicate, on average, how these estimates change with the presence of a recommendation, income and age. Since income and age are mean-centered, their coefficients should be interpreted in terms of deviations of income and age from their mean values - i.e., \$61,000 and 46 years.

== Table 2 ==

The coefficient estimates in the first column of table 2 have reasonable algebraic signs and magnitudes. The trade-up coefficients (κ) are re-parameterized as $\kappa^* = \ln \kappa$ with κ^* estimated without restriction. Thus, negative values of κ^* are synonymous with κ close to zero, indicating a superior offering to those with positive κ^* . We find the premium offerings to have the smallest estimated values of κ , followed by the regular and then the discount offerings with the largest κ . The expenditure for a hypothetical average respondent is, on average, E = $\exp(3.19) = \$24.29$, and the recommendation increases this expenditure to E = $\exp(3.19 + 1.34)$ = \$92.76 for those exposed to it. The expenditure is the maximum amount that a respondent is willing to pay for an offering in the category – offerings above a respondent's threshold level are excluded from the choice set.

A recommendation received for a premium offering changes the perceived superiority (κ^*) as indicated in the second column of coefficients in table 2. Respondents receiving a recommendation view the premium offerings as more superior, and the regular and discount offerings as more inferior than respondents not receiving the recommendation. In addition, Brand G is seen to be viewed as more superior while the other brands degrade in aggregate perceptions of quality, particularly Brand D. Interestingly, the recommendation is seen to have a

negative impact of the perceived quality of attributes A, B and C, and to have a positive impact on expenditure.

The effects of recommendation, income and age on aspects of trade-up explored in more detail in the next section. The presence of the income-age interaction makes it difficult to obtain simple inferences from the Δ coefficients. Moreover, the Δ coefficients are the means of the random-effect distribution, and there exists substantial dispersion of individual-respondent coefficients around these means. Table 3 reports the covariance matrix of random-effects for the coefficients displayed in table 2. The estimates indicated the presence of heterogeneity. Heterogeneity estimates of baseline preference parameters (α) are not reported, but are available from the authors upon request.

== Table 3 ==

5. Describing and Managing Trade-Up

In this section, we examine use of the model for describing and managing trade-up. We draw inferences by computing expected values of the regression model in equation (11). Our goal is to understand the interplay of variables under the control of marketing (recommendation, video) and those that are not (age, income), and we make use of age-income grids maps (see figure 1) for understanding model implications. We use higher color temperatures to indicate higher values of the variable that is displayed. That is, dark blue represents the lowest values with red corresponding to the highest value.

Describing Trade-Up

Figure 2 displays the effects of professional recommendation on the budget expenditure – the maximum amount an individual is willing to pay for an offering in the category. Reported in each cell is the expected expenditure for respondents falling into each age-income group. The left portion of figure 2 displays expected expenditures without the recommendation, and the right portion of the figure displays the expected expenditure after receiving the recommendation. Prior to recommendation, most consumers are willing to spend about \$20, except the younger, high-income group who are willing to spend about \$40. This corresponds to the upper threshold of prices for the regular product. After recommendation, almost all consumers are willing to pay at least \$60 for the product and this corresponds to the lower end of the premium product class.

In both figures, expenditures are highest for younger, high-income respondents and lowest for those that are older and lower-income. This is interesting because it shows that willingness to pay is more related to affluence and disposable income than to the physical need which increases with age. Indeed, it appears that expenditure in this category is, in part, aspirational. Whereas most respondents will pay the necessary \$60-\$80 charge for basic tradeup, younger high income respondents will pay two to three times that amount.

== Figure 2 ==

Figure 3 examines the influence of recommendation on the trade-up parameters for the regular and premium classes of product. Recall that we reparameterized κ to ensure positive values by estimating $\kappa^* = \ln \kappa$ unrestricted, so more negative values of the estimated κ^* correspond to values of κ that are closer to zero, corresponding to offerings that are superior to offerings with positive κ^* . The top portion of figure 3 displays the expected values of κ^* for premium offerings, and the lower portion of figure 3 displays values for regular offerings.

Focusing on the left-hand column "No Recommendation," we see that the premium product is a trade-up for all consumers (i.e., $\kappa^* < 0$), especially the younger, high income group. The regular product, by contrast, is a trade-up only for younger consumers – older consumers who have not received a recommendation, on average, do not perceive a quality distinction between the discount and regular offerings.

The effect of a recommendation on the trade-up parameter is to strengthen the superiority of the premium product but weaken the superiority of the regular product. Receiving a recommendation from a professional is a strong indicator of a consumer's knowledge about the physical problem corresponding to the product category and its remedies, which tends to happen when consumers are older. Awareness of the problem forces a discontinuity in the consumer perception of quality, neutralizing the appeal of products in the \$20 range (regular products) and establishing a price premium for the superior good. This is particularly striking for younger respondents who are the primary consumers of the regular "mid-price" product. Although not many of them are in the process of seeking and receiving advice, those who are make the most dramatic turn-around in perception of the value of the regular product.

== Figure 3 ==

Figure 4 shows the trade up parameter κ^* for two specific items, product 37 and product 40, for respondents not receiving a recommendation. Product 40 is a new entry with limited distribution and Product 37 is a prototype utilizing a new-to-world technology. Both products are at the high end of the range of premium offerings, and the highest priced of all items in the study. Both products were presented to consumers before and after exposure to an ad video. The κ^* values for these products are calculated by summing the levels of the generic attributes corresponding to each item. In particular, these are the only items in the test that contain

Attribute C. Because they are sums, the κ^* values are larger (more negative) and are presented on a different color scale than figure 3.

For all graphs, we notice the familiar pattern of highest trade-up values among younger, higher-income respondents. However, there are interesting subtleties. The κ^* values for Product 40 are banded by age, more so than Product 37, suggesting an age-split in the willingness to pay: younger consumers are more likely to trade-up to Product 40 but older consumers are not, regardless of income. These patterns are then accentuated by the video treatments. For Product 37, whose video was a very technical "how it works" piece, κ^* intensifies among older respondents in the higher income bands. For Product 40, which was more of a "lifestyle" positioning, κ^* intensifies within the younger age band to lower income groups, but diminishes the perceived quality for older respondents.

== Figure 4 ==

Figure 5 shows the same set of graphs but only after receiving a professional recommendation. Recall that in figure 3, the effect of the recommendation was to make κ^* more negative (i.e. improving trade-up) for premium goods. For the general class of premium products, therefore, further knowledge of the problem and its solutions increases willingness to pay. However, adding functionality beyond the premium offering does not necessarily command an additional price premium. Attribute C, which is the key additional function/benefit offered by Products 37 and 40, has positive values of κ^* in table 2 for almost all upper model components and an especially large value for the recommendation component of κ^* (0.92). Therefore further benefits do not lead to increases in perceived quality, and among knowledgeable consumers they can actually reduce the appeal of the offering. For these people, the purchase is probably more imminent and their knowledge more recent; they accept the premium trade-up but need

convincing that even more is even better. Thus, it appears the video diminishes perceived quality for both products for those already receiving the recommendation.

== Figure 5 ==

Managing Trade-Up

The decision to how best to influence the trade-up decision is based on a variety of factors that include the costs of various options and strategic factors that favor specific actions for reasons that are hard to quantify. Our analysis describes two alternatives – a professional recommendation and the use of advertising – as means of influencing trade-up. In our study, the professional recommendation was not experimentally manipulated, and was included in the model as a variable describing the respondent similar to a demographic variable. The video advertisement was experimentally manipulated, and was included in the model as an effect interacting with the focal attribute (attribute C) of the advertisement.

We find from figures 2 and 3 that the recommendation increases expenditure (E) and the perceived quality (κ^*) of premium offerings in the category, while decreasing the perceived quality of regular offerings. Figures 4 and 5 show that the video treatment has different effects for products 37 and 40, and that generally the video has greater effect on respondents who have not received a recommendation. The perceived quality of Product 40 is particularly weak among older respondents, and the video exacerbates this perception. For Product 37, the video improves perceived quality among many of the wealthier respondents.

We quantify the effects of the recommendation and video treatments by calculating expected profits and associated optimal prices for Products 37 and 40 under various conditions. Expected profits are computed as the product of expected demand and contribution margin (i.e., price minus marginal cost). Marginal costs for the products is obtained through consultation with the firm participating in this study, and is estimated to be 36% of regular price. The regular price of Product 37 and Product 40 is \$219.99, and therefore the per-unit marginal cost is \$79.20. Profits are evaluated at optimal prices, which are defined as the profit maximizing prices.

Expected profits and optimal prices are compared using a counter-factual analysis that modifies preferences with and without the recommendation and video effects. The presence and absence of the recommendation is obtained by changing the element of z_i in equation (11) corresponding to the recommendation from zero to one. The other elements of z_i were left unchanged. The presence and absence of the video is obtained by comparing results for the attribute matrix A in equations (9) and (10) to vary from $A = A_1$ to $A = A_{23}$ for all respondents, holding fixed all other variables.

Expected choice probabilities are estimated in each of the four conditions (with and without recommendation, with and without video) by integrating over the distribution of heterogeneity using the unobserved random-effects and observed income and age heterogeneity in the sample:

$$E\left[\Pr\left(x_{k}=1 \mid Data, p, rec, A\right)\right] = \sum_{i=1}^{I} \int \Pr\left(x_{ki}=1 \mid \Delta, V_{\beta}, z_{i}, p, rec, A\right) \pi\left(\Delta, V_{\beta} \mid Data\right) d\Delta \ dV_{\beta}$$
(13)

where "p" indicates the vector of regular prices, "rec" indicates the presence or absence of a recommendation, and "A" is the attribute matrix (either A_1 or A_{23}) in equations (9) and (10). Recall that video (ad) exposure is specified by changing the A matrix. The profits for a one-product firm owning product k are given by

$$\pi_{k}(p_{k}|\Theta) = E\left[\Pr\left(x_{k}=1|price, rec, A, \Theta\right)\right](p_{k}-c_{k})$$
(14)

where Θ refers to the full set of common parameters.

We use the expected choice probabilities along with cost data supplied by the manufacturer to compute expected profits for a given set of product prices. In order to gauge the benefits of the recommendation and ad/video exposure, it is important to consider competitive response. That is, we can't simply exploit the enhanced demand for products derived from the recommendation or ad exposure assuming that the competitor will not change his price. Evaluation of advertising effects in the presence of competition has, heretofore, received little attention in marketing. Counterfactuals are typically constructed which hold the current pricing policy of competitors constant.

We consider the two products (37 and 40) that the videos are targeted at in our equilibrium pricing exercise. We hold the prices of the other brands constant at their current regular prices. Nash equilibrium prices are computed via simultaneous solution of the first order conditions for both firms. Products 37 and 40 are manufactured by different firms so we are considering single product firm equilibrium prices. We note that while multiple equilibria are theoretically possible in our model, we found no evidence of this in our computations. We used both solution of first order conditions (as in 15) as well as alternating best response to a stationary point. Both methods give similar if not identical answers. Numerical solutions to (15) were not sensitive to initial values of price.

$$\frac{\partial \pi_1 \left(p_1 | p_2, \Theta \right)}{\partial p_1} = 0$$

$$\frac{\partial \pi_2 \left(p_2 | p_1, \Theta \right)}{\partial p_2} = 0$$
(15)

Table 4 provides the equilibrium price computations for with and without recommendation and with and without exposure to the video. We also compute the equilibrium per household profits. The results point to the importance of considering competitive effects in pricing in the evaluation of advertising effects. Here exposure to the video and the professional recommendations not only enhance demand for the product advertised but change the structure of competition between the brands.

The recommendation shifts out demand dramatically and this is reflected in prices and profits. We can see that effect of the recommendation is far from uniform but the overall benefits to the manufacturer are very large. If all consumers received a recommendation for the premium products, then profits increase approximately six-fold. The effects of the video exposure are also large but depend on effectiveness of the ad. The video that is designed to promote product 37 has much more information content on the salient feature of this product, while the video for product 40 is more of an "image" ad that extols the lifestyle that is consistent with consumption of the product. The video for product 40 is much less effective than the video for product 37 affording only a 18 per cent increase in profits (conditional on a recommendation) while the video for product 37 increases profits 71 per cent.

While recommendation and video provide an additional source of economic profit which is not fully dissipated by competitive effects, the structure of demand changes as a function of advertising exposure. This can be seen from the equilibrium price computations. As we add video exposure to the recommendation condition (compare the 2nd and 4th rows of table 4),

profits rise but equilibrium prices do not always go up. The optimal price for product 40 actually declines. In our model, which we believe is a more realistic model of demand, the advertising effects the marginal value of quality and this has effects on the structure of demand. These effects play themselves out in non-trivial ways as illustrated in table 4.

== Table 4 ==

6. Conclusion

The non-homothetic utility function was introduced by Allenby and Rossi (1991). Allenby, Shively, Yang and Garratt (2004) considered the non-homothetic model in a study of domestic light beer purchases (i.e., Bud Light, Coors Light and Miller Lite), but found the model comparable in fit to the standard logit model with heterogeneous baseline preferences and price sensitivity.

We demonstrate the benefit of non-homothetic utility for studying trade-up in an extended product category where the prices of offerings differ by more than an order of magnitude. Extended product categories include discount, regular and premium offerings that cannot be considered close substitutes. Our specification of non-homothetic utility differs from previous studies by restricting attention to binary choice data (i.e., $x_k = \{0,1\}$) and introducing a treatment effect (i.e., video) for expenditure. We also find that baseline preferences (α) and trade-up (κ) are empirically identified by the data. The lack of joint identification of α and κ was previously thought to be due to non-empirical reasons. The availability of choice data before and after viewing the video enhanced the information content of the data.

We find large effect sizes for two variables under control of marketers – advertising and professional recommendation. We also find systematic variation in expenditure levels (E) and

views of superiority (κ) across income and age groups. The desire to trade-up is found to be strongest among respondents who are young and wealthy. These individuals have the ability to afford the higher-priced premium offerings, and have exceptionally strong views of their superiority. The results are counter to current practice revealed in the data of targeting recommendations to individuals who are older and wealthy, and suggest there may be an aspiration aspect of demand that warrants further study.

In our non-homothetic demand specification, we are able to incorporate the effect of advertising on the perceived quality or relative superiority of a brand. This means that advertising, properly executed, can increase the perceived quality of a brand and induce a stronger motive to trade-up. In standard homothetic (logit) models, this effect is not possible. There are no trade-up asymmetries in these models. Advertising can only affect the baseline utility or choice probability of a brand but not the rate at which consumers will trade up from brands with lower perceived quality. Advertising is often used to increase the perceived quality of a brand either directly or to accentuate the value to consumers of an attribute of a product. Having the ability of allow advertising to affect the relative superiority of a brand seems to be an important missing component.

A promising avenue for future research is to jointly model the supply-side decision to make professional recommendations. These recommendations were received by respondents prior to start of our study, and we treat them as exogenous variables in our analysis. Professional recommendations, however, are made by health care professionals with different incentives than those of the manufacturer of the product. This means that standard models of firm profit maximization cannot be used to simultaneously model demand for the products and optimal marketing activities. Different models outside the realm of standard assumptions of firm

behavior are required. Manchanda, Chintagunta and Rossi (2004) provide one such approach which is beyond the scope of this paper. Finally, we view this research as calling attention to the need for richer utility specifications appropriate for marketing problems. The field is current dominated by linear utility specifications for reasons of convenience in estimation rather than because homothetic utility is realistic assumption at the consumer level.

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Model	Log Marginal Density
Characteristics Models:	
1. Logit (15 $\tilde{\alpha}$'s)	-28,825
2. Non-Homothetic Logit (15 $\tilde{\alpha}$'s, 15 $\tilde{\kappa}^*$'s)	-25,848
Brand-Specific Models	
3. Logit (40 α's)	-22,487
4. Homothetic Logit (40 α 's, 15 $\tilde{\kappa}^*$'s =e ⁻⁵)	-22,089
5. Non-Homothetic Logit (40 α 's, 15 $\tilde{\kappa}^*$'s)	-21,662

Table 1 Model Fit

Attributes	Intercept	Rec.	Income ¹	Age ²	Inc.xAge
$T_{res} d_{2} U_{res}(\tilde{x}^{*})$					
Brand A	-1.49	0.53	0.09	0.88	-4.52
Diana A	(0.19)	(0.21)	(0.37)	(0.79)	(3.74)
Brand B	-0.86	1.55	0.86	5.88	1.37
	(0.23)	(0.53)	(0.57)	(1.32)	(3.53)
Brand C	0.58	1.46	0.33	3.06	-4.82
	(0.13)	(0.25)	(0.42)	(0.76)	(2.67)
Brand D	-1.01	2.16	(0, 77)	4.76	-1.24
	(0.17)	(0.39)	(0.42)	(0.90) 2 1 2	(3.00)
Brand E	(0.35)	(0.02)	(0 34)	(0 68)	(1 85)
	0.91	1.15	0.33	3.83	-5.80
Brand F	(0.19)	(0.28)	(0.33)	(1.41)	(3.97)
Brand G	0.45	-0.58	1.18	8.61	3.70
Drana O	(0.21)	(0.46)	(0.39)	(1.79)	(5.86)
Discount	1.51	2.66	0.02	8.10	-1.73
	(0.16)	(0.40)	(0.53)	(1.29)	(3.45)
Regular	-0.74	3.15	(0.62)	10.91	(2, 0.0)
	(0.19)	(0.43)	(0.52)	(1.07)	(3.90)
Premium	(0, 20)	(0.27)	(0.32)	(1, 26)	(4, 94)
	(0.20)	(0.27)	(0.02)	(1.20)	(1.51)
A the least of A	-1.69	0.52	0.43	1.22	1.73
Altribute A	(0.17)	(0.21)	(0.31)	(0.80)	(2.22)
Attribute B	-0.75	0.85	-0.63	2.80	3.45
	(0.13)	(0.25)	(0.42)	(0.92)	(2.41)
Attribute C	0.33	0.92	-0.08	4.59	8.69
	(0.30)	(0.53)	(0.59)	(2.00)	(4.79)
Video x Product 37	-0 13	0 67	-1 69	-1 00	-13 49
Interaction	(0.18)	(0, 46)	(0 61)	(1 19)	(5, 18)
Video x Product 40	-0.15	-0.38	-0.75	6.14	-7.15
Interaction	(0.23)	(0.31)	(0.44)	(0.68)	(3.46)
$\frac{1}{1} = \frac{1}{1} = \frac{1}$					
Altordability ($\gamma = \ln E$)	3.19	1.34	0.67	-0.89	-0.63
Outside Good	(0.06)	(0.12)	(0.15)	(0.31)	(1.15)
$(\tau^* = \ln \tau)$	-U.15	-0.53	0.05	-0.80	-0.05
	(0.05)	(0.11)	(∪.⊥3)	(0.32)	(0.98)

 Table 2

 Posterior Means for Selected ∆ Parameters (Posterior Standard Deviation)

¹ Income is in units of \$100,000; ² Age is in units of 100 years.

 $\label{eq:Table 3} \ensuremath{\text{Fosterior Means for Selected V}} V_\beta \ensuremath{\,\text{Parameters}} - Unobserved \ensuremath{\,\text{Heterogeneity}}$

Brand A	7.97																
Brand B	0.9	11.59															
Brand C	1.8	2.57	6.25														
Brand D	0.46	5.58	0.1	8.3													
Brand E	1.49	3.7	0.15	2.22	4.96												
Brand F	0.65	3.73	2.08	1.63	1.79	6.23											
Brand G	-4.09	1.28	-0.23	3.15	-1.09	1.07	12.31										
Discount	-0.69	5.1	3.72	4.94	-0.4	5.99	8.89	21.14									
Regular	1.19	11.32	2.76	6.9	4.6	5.16	2.72	7.56	19.12								
Premium	-0.92	2.39	1.23	0.98	0.72	1.96	-2.08	2.64	2.13	7.06							
Attribute A	0.5	2.65	-0.51	1.95	1.87	0.56	2.43	0.59	2.73	-1.78	5.51						
Attribute B	1.55	3.81	-0.78	1.27	3.23	0.09	-4.09	-5.54	4.3	0.18	1.94	7.74					
Attribute C	-0.14	5.11	2.87	3.83	2.57	3.4	3.24	2.88	3.83	5.27	1.92	0.76	24.17				
Video x SKU37	-1.62	-3.32	0.58	-1.51	-1.94	0.32	1.97	1.28	-3.75	1.32	-1.56	-3.43	5.28	8.55			
Video x SKU40	-0.3	-1.76	1.17	-0.8	-1.59	0.23	-0.27	2.26	-2.17	2.31	-3.19	-2.21	2.35	3.3	5.72		
In(expenditure)	0.53	1.1	0.36	1.15	0.43	-0.31	0.45	1.42	1.3	-0.02	0.69	0.75	0.41	-1.02	-0.17	2.18	
ln(tau)	-0.81	-1.13	-0.81	-0.74	-0.89	-0.73	-0.06	-1.55	-1.52	-0.7	-0.25	-0.41	-2.04	0.06	-0.35	-0.53	1.49

Table 4Equilibrium Prices and Profits (in \$)as a Function of Recommendation and Video

	Equilbriu	um Prices	Equilibrium Profits (per hh)				
	Product 37	Product 40	Product 37	Product 40			
No Recommendation, No Video	185.4	194.5	.45	.64			
Recommendation No Video	285.2	394.1	2.89	4.44			
No Recommendation Video	191.5	236.3	.78	.66			
Recommendation Video	472.5	302.2	4.93	5.24			

Figure 1 Respondent Counts and Percent Receiving Recommendation

Counts



Recommendation



Figure 2 Effect of Recommendation on Expenditure

No Recommendation



Recommendation



 $\begin{array}{c} \mbox{Figure 3} \\ \mbox{Effect of Recommendation on Trade-Up} \left(\kappa^{*} \right) \mbox{for} \\ \mbox{Premium and Regular Offerings} \end{array}$

		Income Midpoint										
		7.5	20	27.5	35	45	62.5	87.5	125			
	25	-1.72	-1.83	-1.89	-1.96	-2.04	-2.19	-2.40	-2.72			
	30	-1.59	-1.70	-1.76	-1.82	-1.91	-2.05	-2.26	-2.58			
¥	35	-1.46	-1.56	-1.63	-1.69	-1.77	-1.92	-2.12	-2.44			
Age Midpoin	40	-1.33	-1.43	-1.49	-1.55	-1.64	-1.78	-1.99	-2.30			
	45	-1.19	-1.30	-1.36	-1.42	-1.50	-1.64	-1.85	-2.15			
	50	-1.06	-1.16	-1.22	-1.28	-1.37	-1.51	-1.71	-2.01			
	55	-0.93	-1.03	-1.09	-1.15	-1.23	-1.37	-1.57	-1.87			
	60	-0.80	-0.90	-0.96	-1.02	-1.10	-1.23	-1.43	-1.73			
	65	-0.66	-0.76	-0.82	-0.88	-0.96	-1.10	-1.30	-1.59			
	70	-0.53	-0.63	-0.69	-0.75	-0.82	-0.96	-1.16	-1.45			

No Recommendation



Recommendation

		Income Midpoint										
		7.5	20	27.5	35	45	62.5	87.5	125			
	25	0.28	0.24	0.22	0.20	0.17	0.11	0.04	-0.07			
	30	0.71	0.70	0.69	0.68	0.68	0.66	0.64	0.61			
÷	35	1.14	1.15	1.16	1.17	1.19	1.21	1.24	1.29			
poin	40	1.57	1.61	1.64	1.66	1.70	1.76	1.85	1.98			
Mid	45	2.00	2.07	2.11	2.15	2.21	2.31	2.45	2.66			
Age	50	2.42	2.52	2.58	2.64	2.72	2.86	3.05	3.35			
	55	2.85	2.98	3.05	3.13	3.23	3.40	3.65	4.03			
	60	3.28	3.44	3.53	3.62	3.74	3.95	4.26	4.71			
	65	3.71	3.89	4.00	4.11	4.25	4.50	4.86	5.40			
	70	4.14	4.35	4.47	4.60	4.76	5.05	5.46	6.08			

		Income Midpoint										
		7.5	20	27.5	35	45	62.5	87.5	125			
	25	-2.88	-2.91	-2.94	-2.96	-2.99	-3.04	-3.12	-3.23			
	30	-2.45	-2.46	-2.46	-2.47	-2.48	-2.49	-2.51	-2.54			
t.	35	-2.02	-2.00	-1.99	-1.98	-1.97	-1.94	-1.91	-1.86			
poin	40	-1.59	-1.54	-1.52	-1.49	-1.46	-1.40	-1.31	-1.18			
Mid	45	-1.16	-1.09	-1.05	-1.00	-0.95	-0.85	-0.71	-0.49			
Age	50	-0.73	-0.63	-0.57	-0.51	-0.44	-0.30	-0.10	0.19			
	55	-0.30	-0.18	-0.10	-0.03	0.07	0.25	0.50	0.88			
	60	0.13	0.28	0.37	0.46	0.59	0.80	1.10	1.56			
	65	0.56	0.74	0.84	0.95	1.10	1.35	1.71	2.24			
	70	0.99	1.19	1.32	1.44	1.61	1.89	2.31	2.93			

Figure 4 Effect of Video on Trade-Up (κ^*) for Attribute C for Respondents <u>Not</u> Receiving a Recommendation





Product 40





Video

No Video

Figure 5 Effect of Video on Trade-Up (κ^*) for Attribute C For Respondents Receiving a Recommendation





Product 40

Income Midpoint 7.5 20 27.5 35 45 62.5 87.5 125 -5.47 -3.98 -4.22 -4.35 -4.49 -4.68 -5.01 -6.17 25 -3.31 -3.57 -3.73 -3.88 -4.09 -4.45 -4.96 -5.73 30 35 -2.65 -2.93 -3.10 -3.27 -3.49 -3.89 -4.45 -5.30 Age Midpoint -2.29 -2.47 -2.65 -2.90 -3.33 -3.95 -4.87 40 -1.98 -1.31 -1.64 -1.84 -2.04 -2.31 -2.77 -3.44 -4.43 45 50 -1.21 -1.43 -1.71 -2.21 -2.93 -4.00 -0.64 -1.00 55 0.03 -0.36 -0.59 -0.82 -1.12 -1.66 -2.42 -3.57 0.29 -1.10 -1.91 -3.13 60 0.69 0.04 -0.20 -0.53 -0.54 -1.40 -2.70 1.36 0.93 0.67 0.41 0.07 65 -0.90 -2.27 70 2.03 1.57 1.30 1.02 0.66 0.02

Income Midpoint 7.5 20 27.5 35 45 62.5 87.5 125 -6.80 -7.16 -7.38 -7.60 -7.89 -8.40 25 -6.08 -5.78 -6.25 -6.43 -6.67 -7.09 30 35 -4.76 -4.99 -5.13 -5.27 -5.45 -5.77 -6.23 -6.93 Age Midpoint -3.74 -3.90 -4.00 -4.10 -4.23 -4.46 -4.79 -5.28 40 45 -2.72 -2.82 -2.87 -2.93 -3.01 -3.15 -3.35 -3.64 -1.73 50 -1.70 -1.75 -1.77 -1.79 -1.84 -1.90 -2.00 55 -0.67 -0.64 -0.62 -0.60 -0.57 -0.53 -0.46 -0.36 0.45 0.51 0.57 0.79 0.99 1.29 60 0.35 0.65 1.53 2.10 1.37 1.73 2.43 65 1.63 1.87 2.93 70 2.39 2.62 2.76 2.90 3.09 3.41 3.87 4.57